As Level Mathematics

Function

Pure Mathematics I

2023-2025 Series

Name:

CLASS:

Cambridge Assessment International Education

Cambridge International School

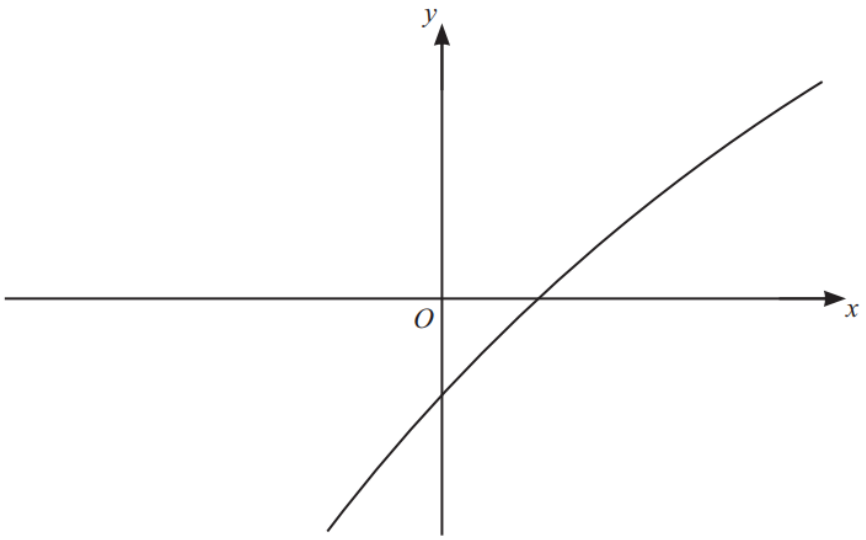
1. 9709_s25_qp_11 no 10

The functions f and g are defined by

$$f(x) = \sqrt{x} \qquad \text{for } x \geqslant 0,$$

$$g(x) = 3\sqrt{x+2} - 5 \qquad \text{for } x \geqslant -2.$$

(a) Describe fully a sequence of transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. You should make clear the order in which the transformations are applied. [5]



The diagram shows the graph of $y = g(x)$.

(b) On the diagram sketch the graph of $y = g^{-1}(x)$ together with any relevant mirror line. [2]

(c) Find an expression for $g^{-1}(x)$. [2]

(d) State the range of g^{-1}. [1]

The function h is defined by

$$h(x) = x - 2 \qquad \text{for } x \geqslant 0.$$

(e) Find the value of $g^{-1}h(4)$. [1]

2. 9709_s25_qp_13 no 11

The function f is defined by $f(x) = x^2 + 4ax + a$ for $x \in \mathbb{R}$, where a is a constant.

The function g is such that $g^{-1}(x) = \sqrt[3]{2x-4}$ for $x \in \mathbb{R}$.

(a) Given that the range of f is $f(x) \geqslant -33$, find the possible values of a. [4]

(b) Given instead that $fgg(0) = 96$, find the value of a. [6]

3. 9709_s25_qp_15 no 9

Functions f and g are defined as follows.

$$f(x) = \cos x \text{ for } 0 \leqslant x \leqslant \pi$$
$$g(x) = 3\cos(x - \pi) + 2 \text{ for } \pi \leqslant x \leqslant 2\pi$$

(a) Describe fully the transformations that have been combined to transform the graph of $y = f(x)$ to the graph of $y = g(x)$. [4]

(c) Find $g^{-1}f\left(\frac{1}{3}\pi\right)$. [4]

4. 9709_m25_qp_12 no 11

Functions f and g are defined for all real values of x by

$$f(x) = 4x^2 - c \quad \text{and} \quad g(x) = 2x + k,$$

where c and k are positive constants. It is given that $g^{-1}(3k+1) = c$.

(a) Show that $gf(x) = 8x^2 - k - 1$. [4]

(b) The curve with equation $y = 8x^2 - k - 1$ is transformed to the curve with equation $y = h(x)$ by the following sequence of transformations.

Translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
Stretch in the y-direction by scale factor k
Reflection in the x-axis

Find an expression for $h(x)$ in terms of x and k. [3]

(c) The range of h is given by $h(x) \leqslant 15$.

Find the values of c and k. [3]

5. 9709_w24_qp_11 no 11

The function f is defined by $f(x) = 3+6x-2x^2$ for $x \in \mathbb{R}$.

(a) Express $f(x)$ in the form $a-b(x-c)^2$, where a, b and c are constants, and state the range of f. [3]

(b) The graph of $y = f(x)$ is transformed to the graph of $y = h(x)$ by a reflection in one of the axes followed by a translation. It is given that the graph of $y = h(x)$ has a minimum point at the origin.

Give details of the reflection and translation involved. [2]

The function g is defined by $g(x) = 3+6x-2x^2$ for $x \leqslant 0$.

(c) Sketch the graph of $y = g(x)$ and explain why g is a one-one function. You are **not** required to find the coordinates of any intersections with the axes. [2]

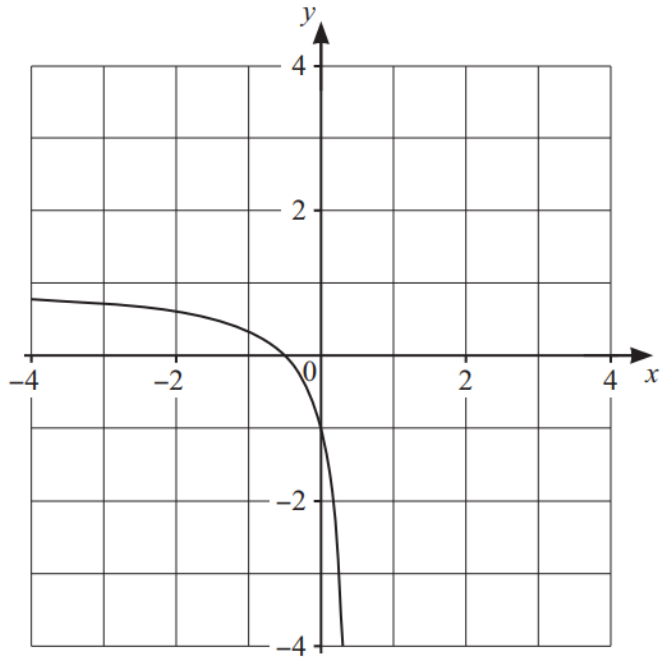
(d) Sketch the graph of $y = g^{-1}(x)$ on your diagram in **(c)**, and find an expression for $g^{-1}(x)$. You should label the two graphs in your diagram appropriately and show any relevant mirror line. [4]

6. 9709_w24_qp_12 no 5

The function f is defined by $f(x) = \frac{2x+1}{2x-1}$ for $x < \frac{1}{2}$.

(a) (i) State the value of $f(-1)$. [1]

(ii)



The diagram shows the graph of $y = f(x)$. Sketch the graph of $y = f^{-1}(x)$ on this diagram. Show any relevant mirror line. [2]

(iii) Find an expression for $f^{-1}(x)$ and state the domain of the function f^{-1}. [4]

The function g is defined by $g(x) = 3x+2$ for $x \in \mathbb{R}$.

(b) Solve the equation $f(x) = gf\left(\frac{1}{4}\right)$. [3]

7. 9709_w24_qp_13 no 8

(a) Express $3x^2 - 12x + 14$ in the form $3(x+a)^2 + b$, where a and b are constants to be found. [2]

The function $f(x) = 3x^2 - 12x + 14$ is defined for $x \geqslant k$, where k is a constant.

(b) Find the least value of k for which the function f^{-1} exists. [1]

For the rest of this question, you should assume that k has the value found in part **(b)**.

(c) Find an expression for $f^{-1}(x)$. [3]

(d) Hence or otherwise solve the equation $ff(x) = 29$. [3]

8. 9709_m24_qp_12 NO 9

The functions f and g are defined for all real values of x by

$$\mathrm{f}(x) = (3x-2)^2 + k \quad \text{and} \quad \mathrm{g}(x) = 5x - 1,$$

where k is a constant.

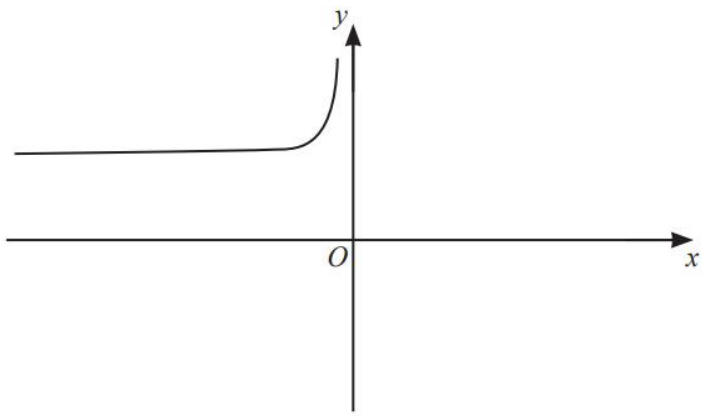
(a) Given that the range of the function gf is $\mathrm{gf}(x) \geqslant 39$, find the value of k. [4]

(b) For this value of k, determine the range of the function fg. [2]

(c) The function h is defined for all real values of x and is such that $\mathrm{gh}(x) = 35x + 19$.

Find an expression for $\mathrm{g}^{-1}(x)$ and hence, or otherwise, find an expression for $\mathrm{h}(x)$. [3]

9. 9709_s24_qp_11 no 6



The function f is defined by $f(x) = \frac{2}{x^2} + 4$ for $x < 0$. The diagram shows the graph of $y = f(x)$.

(a) On this diagram, sketch the graph of $y = f^{-1}(x)$. Show any relevant mirror line. [2]

(b) Find an expression for $f^{-1}(x)$. [3]

(c) Solve the equation $f(x) = 4.5$. [1]

(d) Explain why the equation $f^{-1}(x) = f(x)$ has no solution. [1]

10. 9709_s24_qp_12 no 2

The curve $y = x^2$ is transformed to the curve $y = 4(x-3)^2 - 8$.

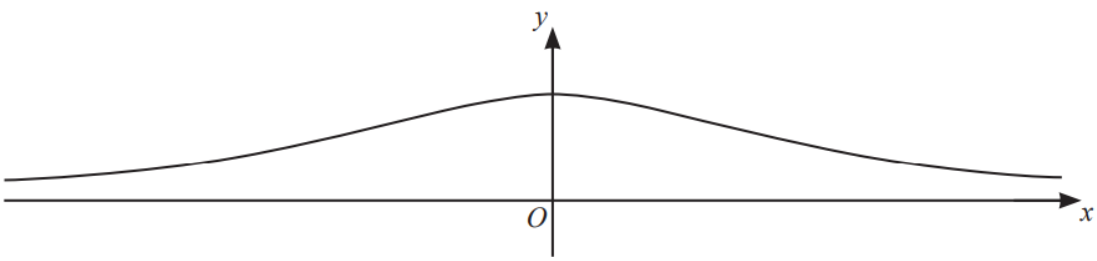
Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations have been applied. [5]

11. 9709_s24_qp_12 no 4

The function f is defined as follows:

$$f(x) = \sqrt{x} - 1 \text{ for } x > 1.$$

(a) Find an expression for $f^{-1}(x)$. [1]



The diagram shows the graph of $y = g(x)$ where $g(x) = \frac{1}{x^2+2}$ for $x \in \mathbb{R}$.

(b) State the range of g and explain whether g^{-1} exists. [2]

The function h is defined by $\mathrm{h}(x) = \dfrac{1}{x^2+2}$ for $x \geqslant 0$.

(c) Solve the equation $\mathrm{hf}(x) = \mathrm{f}\left(\dfrac{25}{16}\right)$. Give your answer in the form $a+b\sqrt{c}$, where a, b and c are integers. [4]

12. 9709_s24_qp_13 no 11

The function f is defined by $\mathrm{f}(x) = 10+6x-x^2$ for $x \in \mathbb{R}$.

(a) By completing the square, find the range of f. [3]

The function g is defined by $\mathrm{g}(x) = 4x+k$ for $x \in \mathbb{R}$ where k is a constant.

(b) It is given that the graph of $y = \mathrm{g}^{-1}\mathrm{f}(x)$ meets the graph of $y = \mathrm{g}(x)$ at a single point P.

Determine the coordinates of P. [6]

13. 9709_w23_qp_11 no 4

The transformation R denotes a reflection in the x-axis and the transformation T denotes a translation of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

(a) Find the equation, $y = \mathrm{g}(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations R followed by T. [2]

(b) Find the equation, $y = \mathrm{h}(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations T followed by R. [2]

(c) State fully the transformation that maps the curve $y = \mathrm{g}(x)$ onto the curve $y = \mathrm{h}(x)$. [2]

14. 9709_w23_qp_11 no 9

(a) Express $4x^2 - 12x + 13$ in the form $(2x + a)^2 + b$, where a and b are constants. [2]

The function f is defined by $\mathrm{f}(x) = 4x^2 - 12x + 13$ for $p < x < q$, where p and q are constants. The function g is defined by $\mathrm{g}(x) = 3x + 1$ for $x < 8$.

(b) Given that it is possible to form the composite function gf, find the least possible value of p and the greatest possible value of q. [3]

(c) Find an expression for $\mathrm{gf}(x)$. [1]

The function h is defined by $\mathrm{h}(x) = 4x^2 - 12x + 13$ for $x < 0$.

(d) Find an expression for $\mathrm{h}^{-1}(x)$. [3]

15. 9709_w23_qp_12

The equation of a curve is $y = x^2 - 8x + 5$.

(a) Find the coordinates of the minimum point of the curve. [2]

The curve is stretched by a factor of 2 parallel to the y-axis and then translated by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

(b) Find the coordinates of the minimum point of the transformed curve. [2]

(c) Find the equation of the transformed curve. Give the answer in the form $y = ax^2 + bx + c$, where a, b and c are integers to be found. [4]

16. 9709_w23_qp_12 no 8

Functions f and g are defined by

$$\text{f}(x) = (x + a)^2 - a \text{ for } x \leqslant -a,$$
$$\text{g}(x) = 2x - 1 \text{ for } x \in \mathbb{R},$$

where a is a positive constant.

(a) Find an expression for $\text{f}^{-1}(x)$. [3]

(b) **(i)** State the domain of the function f^{-1}. [1]

(ii) State the range of the function f^{-1}. [1]

(c) Given that $a = \frac{7}{2}$, solve the equation $\text{gf}(x) = 0$. [3]

17. 9709_w23_qp_13 no 7

The function f is defined by $f(x) = 1 + \frac{3}{x-2}$ for $x > 2$.

(a) State the range of f. [1]

(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

The function g is defined by $g(x) = 2x - 2$ for $x > 0$.

(c) Obtain a simplified expression for $gf(x)$. [2]

18. 9709_m23_qp_12 no 2

A function f is defined by $f(x) = x^2 - 2x + 5$ for $x \in \mathbb{R}$. A sequence of transformations is applied in the following order to the graph of $y = f(x)$ to give the graph of $y = g(x)$.

Stretch parallel to the x-axis with scale factor $\frac{1}{2}$

Reflection in the y-axis

Stretch parallel to the y-axis with scale factor 3

Find $g(x)$, giving your answer in the form $ax^2 + bx + c$, where a, b and c are constants. [4]

19. 9709_m23_qp_12 no 9

The function f is defined by $f(x) = -3x^2 + 2$ for $x \leqslant -1$.

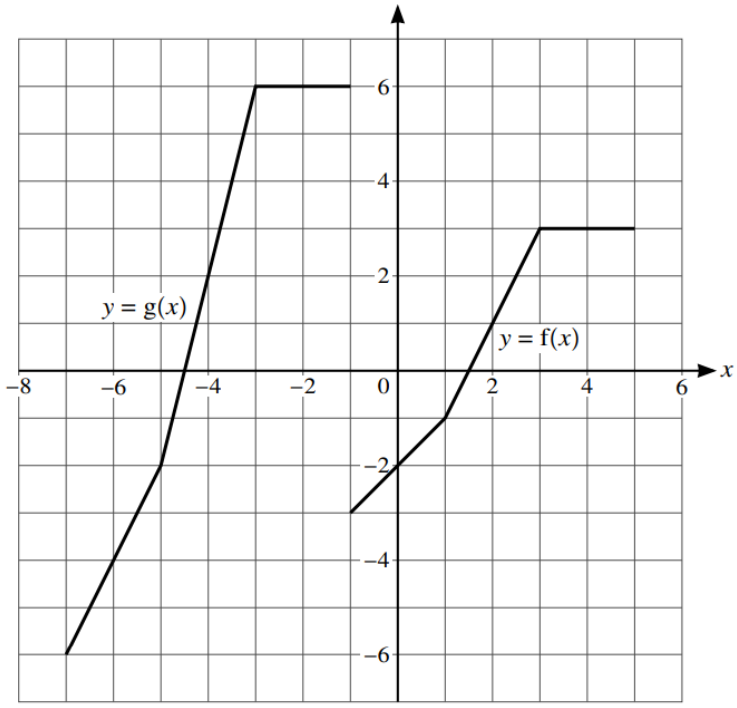
(a) State the range of f. [1]

(b) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g(x) = -x^2 - 1$ for $x \leqslant -1$.

(c) Solve the equation $fg(x) - gf(x) + 8 = 0$. [5]

20. 9709_s23_qp_11 no 3



The diagram shows graphs with equations $y = f(x)$ and $y = g(x)$.

Describe fully a sequence of two transformations which transforms the graph of $y = f(x)$ to $y = g(x)$. [4]

21. 9709_s23_qp_11 no 8

The functions f and g are defined as follows, where a and b are constants.

$$\mathrm{f}(x) = 1 + \frac{2a}{x-a} \text{ for } x > a$$

$$\mathrm{g}(x) = bx - 2 \text{ for } x \in \mathbb{R}$$

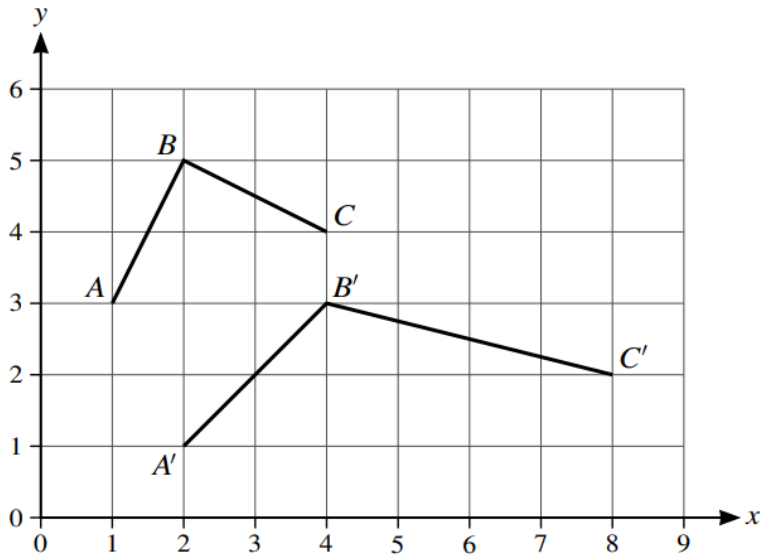
(a) Given that $\mathrm{f}(7) = \frac{5}{2}$ and $\mathrm{gf}(5) = 4$, find the values of a and b. [4]

For the rest of this question, you should use the value of a which you found in **(a)**.

(b) Find the domain of f^{-1}. [1]

(c) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

22. 9709_s23_qp_13 no 1



The diagram shows the graph of $y = \mathrm{f}(x)$, which consists of the two straight lines AB and BC. The lines $A'B'$ and $B'C'$ form the graph of $y = \mathrm{g}(x)$, which is the result of applying a sequence of two transformations, in either order, to $y = \mathrm{f}(x)$.

State fully the two transformations. [4]

23. 9709_s23_qp_13 no 7

The function f is defined by $f(x) = 2 - \frac{5}{x+2}$ for $x > -2$.

(a) State the range of f. [1]

(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

The function g is defined by $g(x) = x + 3$ for $x > 0$.

(c) Obtain an expression for $fg(x)$ giving your answer in the form $\frac{ax+b}{cx+d}$, where a, b, c and d are integers. [3]